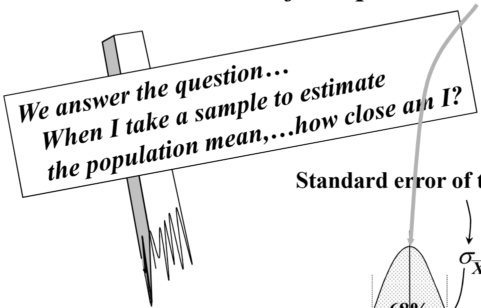
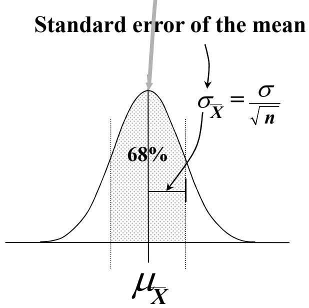
Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

Distribution of sample means.





$$\mu_{\bar{X}} = \mu$$
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

68% of the area is within ($\mu \pm 1\sigma_{\bar{X}}$)

95% of the area is within ($\mu \pm 2\sigma_{\bar{X}}$)

99.7% of the area is within ($\mu \pm 3\sigma_{\bar{X}}$)

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

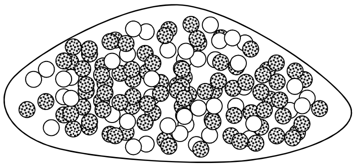
Estimation

Point Estimate:
A single number which uses sample information to estimate the value of a population parameter.

Interval Estimate:
An estimate of the range of values within which the population parameter is likely to fall.

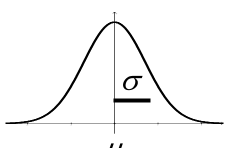
Estimation Overview

Consider a Population



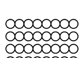
Describe the Population using

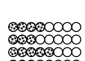
μ and σ or p



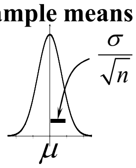
p

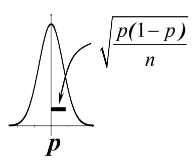
take samples





sample means





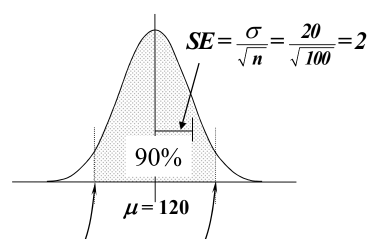
Confidence Intervals for the Population Mean

Consider a population of 20-year old men and women.

Blood Pressure: $\mu = 120$ $\sigma = 20$

Take repeated samples of size $n = 100$

Distribution of sample means is normal



Recall: For a normal distribution, 90% of the data falls between z = –1.64 and z = +1.64.

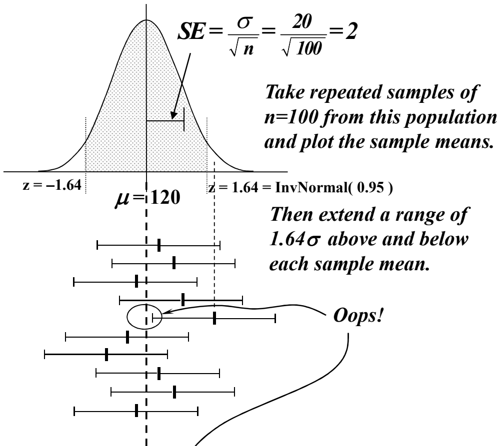
Confidence Intervals for the Population Mean

Consider a population of 20-year old men and women.

Blood Pressure: $\mu = 120$ $\sigma = 20$

Take repeated samples of size $n = 100$

Distribution of sample means is normal



For 90% of the sample means, the range extended about the sample mean will include the true mean of the population, μ. Thus 90% is the CONFIDENCE LEVEL.

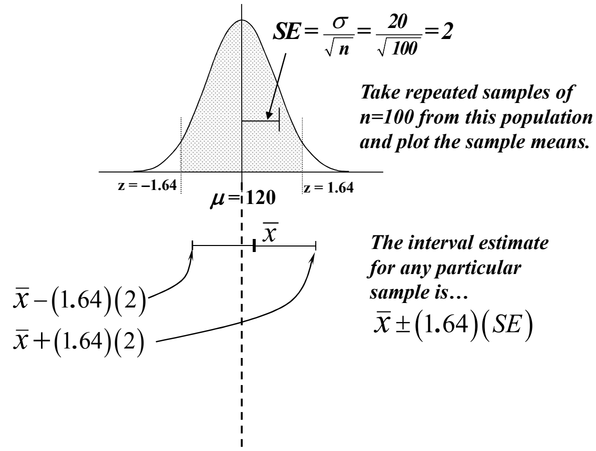
Confidence Intervals for the Population Mean

Consider a population of 20-year old men and women.

Blood Pressure: $\mu = 120$ $\sigma = 20$

Take repeated samples of size $n = 100$

Distribution of sample means is normal



This range of values is called the 90% CONFIDENCE INTERVAL

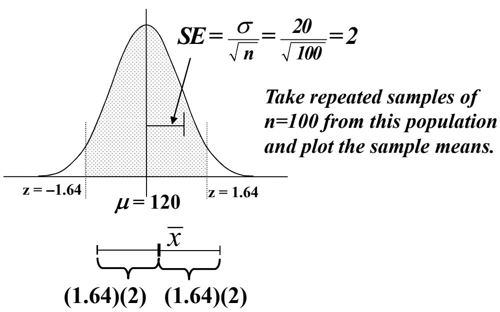
Confidence Intervals for the Population Mean

Consider a population of 20-year old men and women.

Blood Pressure: $\mu = 120$ $\sigma = 20$

Take repeated samples of size $n = 100$

Distribution of sample means is normal



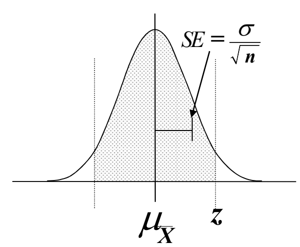
To determine the population mean, μ, at a 90% Confidence Level, take a sample and determine its mean, $\bar{x}$.

There is a 90% probability that μ will be in the range

$$\bar{x} - (1.64)(SE) < \mu < \bar{x} + (1.64)(SE)$$

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

Other Confidence Levels



Confidence Level	*z-score*	*Confidence Interval*
90%	***1.64***	$\bar{x} \pm 1.64 \frac{\sigma}{\sqrt{n}}$
95%	***1.96***	$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$
99%	***2.58***	$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$

Margin of Error

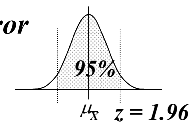
Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

To estimate the mean amount overdue in all its delinquent accounts, a bank randomly samples 49 accounts and finds the sample mean, $\bar{x}$, to be $237.

Based on past history, the bank uses a standard deviation, σ, of $53.

1. Determine the 95% Confidence Interval for the mean amount overdue.

2. Determine the Margin of Error



For a 95% Confidence Level, z = InvNormal(.975) = 1.96

Since σ = $53 and n = 49,

the Standard Error of the Mean $= \frac{\sigma}{\sqrt{n}} = \frac{53}{\sqrt{49}} = 7.57$

Since the Sample Mean was $237, the 95% Confidence Interval is

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z\frac{\sigma}{\sqrt{n}}$$

$$\$237 - (1.96)(7.57) < \mu < \$237 + (1.96)(7.57)$$

$$\$222.16 < \mu < \$251.84$$

$14.84 Margin of Error

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

To repeat the problem using the calulator:

$\bar{x}$ = $237 σ = $53 n = 49

STATS ➡ **TESTS** ➡ **ZInterval**

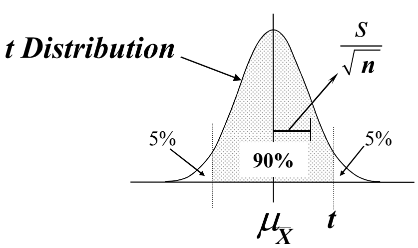
```
ZInterval
 Inpt:Data Stats
 σ:53
 x̄:237
 n:49
 C-Level:.95
 Calculate
```

```
ZInterval
 (222.16,251.84)
 x̄=237
 n=49
```

Margin of Error = 251.84 - 237 = 14.84

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

Distribution of sample means.
σ is unknown



s is the sample standard deviation (determined from the sample)

To find, say the 90% Confidence Interval, determine the t-score which encloses 90% of the sample means.

Once the t-score ($t_{95\%}$)is found,

------------ 90% CONFIDENCE INTERVAL ------------

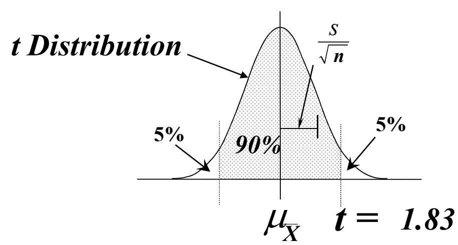
$$\bar{x} - t_{95\%}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{95\%}\frac{s}{\sqrt{n}}$$ ***with 90% probability***

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

Distribution of sample means.
σ is unknown

To find the t-scores when you know the area under the t-distribution, you must use a "t-table."

For example, to know the t-scores that enclose 90% of the area in the center of the t-distribution using a sample size of n = 10,



Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

To find the t-score when you know the area under the t-distribution, you must use a "t-table."

To find $t_{95\%}$

Degrees of Freedom (df) is simply (n -1)

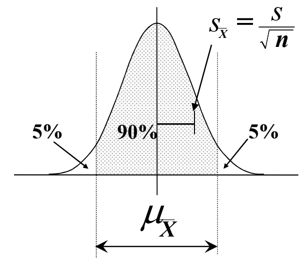
…for n = 10, df = 9 t = 1.83

Appendix D Table III Page 734

degrees of freedom	ONE TAIL critical t left (t_c)	ONE TAIL critical t right (t_c)	TWO TAIL critical t left (t_{LC})	TWO TAIL critical t right (t_{RC})	ONE TAIL critical t left (t_c)	ONE TAIL critical t right (t_c)
FOR CONFIDENCE INTERVALS						
df	$t_{01\%}$	$t_{99\%}$	$t_{0.5\%}$	$t_{99.5\%}$	$t_{5\%}$	$t_{95\%}$
1	−31.82	31.82	−63.66	63.66	−6.31	6.31
2	−6.96	6.96	−9.92	9.92	−2.92	2.92
3	−4.54	4.54	−5.84	5.84	−2.35	2.35
4	−3.75	3.75	−4.60	4.60	−2.13	2.13
5	−3.36	3.36	−4.03	4.03	−2.02	2.02
6	−3.14	3.14	−3.71	3.71	−1.94	1.94
7	−3.00	3.00	−3.50	3.50	−1.90	1.90
8	−2.90	2.90	−3.36	3.36	−1.86	1.86
9	−2.82	2.82	−3.25	3.25	−1.83	1.83
10	−2.76	2.76	−3.17	3.17	−1.81	1.81
11	−2.72	2.72	−3.11	3.11	−1.80	1.80
12	−2.68	2.68	−3.06	3.06	−1.78	1.78
13	−2.65	2.65	−3.01	3.01	−1.77	1.77

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

The t-Distribution



Confidence Level	*Confidence Interval*
90%	$\bar{x} - t_{95\%}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{95\%}\frac{s}{\sqrt{n}}$
95%	$\bar{x} - t_{97.5\%}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{97.5\%}\frac{s}{\sqrt{n}}$
99%	$\bar{x} - t_{99.5\%}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{99.5\%}\frac{s}{\sqrt{n}}$

Section 9.4: Interval Estimation:
Confidence Intervals for the Population Mean

Page 547 Example 8.3

English professor estimates the number of typing errors per page in term papers by collecting a sample of 36 papers.

From this sample, she finds a mean of 4.6 errors per page with s = 1.4. What is the 90% confidence interval for all term papers?

Sample Std Dev $s = 1.4$

Mean Errors/Page $\bar{x} = 4.6$

Sample Size $n = 36$

$df = 36 - 1 = 35$

For 90% Confidence Interval,

$$\bar{x} - (t_{95\%})\frac{s}{\sqrt{n}} \quad to \quad \bar{x} + (t_{95\%})\frac{s}{\sqrt{n}}$$

$$4.6 - 1.69\frac{1.4}{\sqrt{36}} \quad to \quad 4.6 + 1.69\frac{1.4}{\sqrt{36}}$$

$$4.206 \quad to \quad 4.994$$

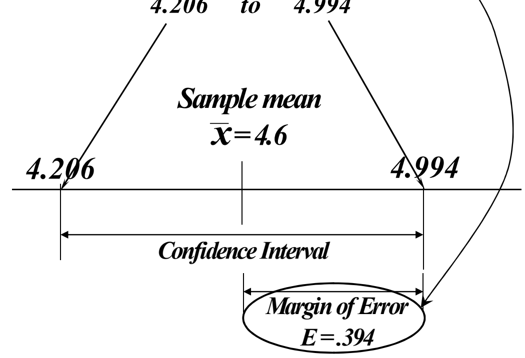
Section 9.6: Determining Sample Size and Margin of Error

For 90% Confidence Interval,

$$\bar{x} - (t_{95\%})\frac{s}{\sqrt{n}} \quad to \quad \bar{x} + (t_{95\%})\frac{s}{\sqrt{n}}$$

$$4.6 - 1.69\frac{1.4}{\sqrt{36}} \quad to \quad 4.6 + 1.69\frac{1.4}{\sqrt{36}}$$

$$4.206 \quad to \quad 4.994$$



Interval Estimation Using the Calculator

To estimate the Summer traffic across the Throgs Neck Bridge, the DOT sampled the traffic on 10 randomly selected Summer days. In thousands of cars, the results were…

112	*115*	*139*	*128*	*130*
122	*145*	*132*	*102*	*136*

thousand cars per day.

Use the calculator to estimate the mean daily traffic and the Margin of Error for the daily Summer traffic at a 90%, 95% and 99% Confidence Level.

STATS → **TESTS** → **TInterval**

```
TInterval
 Inpt:Data Stats
 List:L1
 Freq:1
 C-Level:.9
 Calculate
```

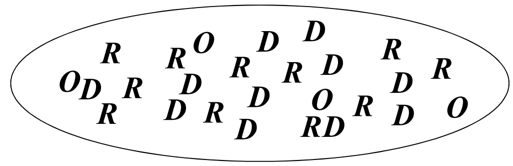
```
TInterval
 (118.38,133.82)
 x̄=126.1
 Sx=13.31206637
 n=10
```

At a 90% Confidence Level

Est Mean Traffic = 126.1
Margin of Error = 7.72

Section 9.5: Interval Estimation: <u>Population Proportion</u>

In the population there are millions of Republicans, Democrats and Others.



The proportion of Republicans in the <u>population</u> is

$$p_R = \frac{\textit{Number of Republicans in the population}}{\textit{Total population}} = \frac{N_R}{N}$$

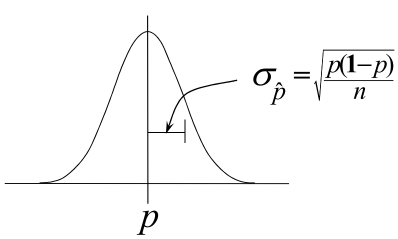
If we take a <u>sample</u> of n people from the population, the proportion of Republicans in the sample will be

$$\hat{p}_R = \frac{\textit{Number of Republicans in the sample}}{\textit{Sample size}} = \frac{x}{n}$$

Section 9.5: Interval Estimation: <u>Population Proportion</u>

In general $\hat{p}_R$ (sample proportion) would NOT equal = p_R (population proportion).

If we took repeated samples from the population and determined the proportions from each sample, they would form a normal distribution.



The mean of the distribution of the proportion is p and the standard error of the proportion is

$$\textit{Std Error of Prop} = \sqrt{\frac{p(1-p)}{n}}$$

Section 9.5: Interval Estimation: <u>Population Proportion</u>

When we are estimating the population proportion, p, from the sample proportion, $\hat{p}$, we use $s_{\hat{p}}$ to represent the standard error of the proportion.

$$s_{\hat{p}} = \sqrt{\frac{\hat{p}\,(1 - \hat{p})}{n}}$$

Therefore, the confidence interval of a proportion is

90%: $\hat{p} - (1.65)s_{\hat{p}} < p < \hat{p} + (1.65)s_{\hat{p}}$

95%: $\hat{p} - (1.96)s_{\hat{p}} < p < \hat{p} + (1.96)s_{\hat{p}}$

99%: $\hat{p} - (2.58)s_{\hat{p}} < p < \hat{p} + (2.58)s_{\hat{p}}$

Section 9.5: Interval Estimation: <u>Population Proportion</u>

From a semester's class survey of 35 students, 30 students indicated that they believed in God.

From this sample what is the 90% confidence interval for the proportion of all students at Nassau who believe in God.

From the sample

$$\text{Sample proportion} = \hat{p} = \frac{\text{believers}}{\text{sample size}} = \frac{30}{35} = 0.857$$

Standard error of the proportion: $= s_{\hat{p}}$

$$s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.857(1-0.857)}{35}} = .059$$

Use Normal Distribution for Confidence Intervals

$$90\%: \quad \hat{p} - (1.65)s_{\hat{p}} < p < \hat{p} + (1.65)s_{\hat{p}}$$

$$95\%: \quad \hat{p} - (1.96)s_{\hat{p}} < p < \hat{p} + (1.96)s_{\hat{p}}$$

$$99\%: \quad \hat{p} - (2.58)s_{\hat{p}} < p < \hat{p} + (2.58)s_{\hat{p}}$$

Chapter 9 - Interval Estimation

Section 9.5: Interval Estimation: <u>Population Proportion</u>

$$\text{So } \hat{p} = \frac{\text{believers}}{\text{sample size}} = 0.857 \quad \text{and} \quad s_{\hat{p}} = .059$$

then for a confidence level of 90%:

90%:

$\hat{p} - (1.65)s_{\hat{p}}$ *to* $\hat{p} + (1.65)s_{\hat{p}}$

0.857 - (1.65)(.059) to 0.857 + (1.65)(.059)

<u>0.760 to 0.954</u>

95%:

$\hat{p} - (1.96)s_{\hat{p}}$ *to* $\hat{p} + (1.96)s_{\hat{p}}$

0.857 - (1.96)(.059) to 0.857 + (1.96)(.059)

<u>0.741 to 0.973</u>

99%:

$\hat{p} - (2.56)s_{\hat{p}}$ *to* $\hat{p} + (2.56)s_{\hat{p}}$

0.857 - (2.56)(.059) to 0.857 + (2.56)(.059)

<u>0.706 to 1.008</u>

Chapter 9 - Interval Estimation

Section 9.6: Determining Sample Size and Margin of Error

To estimate the mean flight time between two cities, a sample of 64 flights during the year was taken. The sample had a mean of 2 hours and a sample standard deviation, s, of 20 minutes.

At a confidence level of 95%, what is the confidence interval for all flight times between those cities? What is the margin of error?

Since σ is unknown, we use TInterval with …

$$\bar{x} = 120 \text{ minutes},$$

$$s_x = 20 \text{ minutes},$$

$$n = 64$$

$$\text{and} \quad \text{C-Level} = .95$$

The TInterval is (115, 125) minutes.

So the Margin of Error

E = (125 - 115) / 2 = 5 minutes

Chapter 9 - Interval Estimation

Overview

Estimate Pop Mean	**Estimate Pop Prop**
μ	p
e.g. age, grade, weight, income, days of gestation, etc.	*e.g. prop of Republicans, prop of 60 year olds, prop of females, etc.*

Take sample of size n and decide on confidence level e.g., 90%, 95% or 99%

Determine $\bar{x}$ and s.	**Determine $\hat{p}$ = x/n.**
t-distribution with df = n-1	*normal distribution if np>5 and n(1-p)>5*
$s_{\bar{x}} = s / \sqrt{n}$	$s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$\bar{x} \pm (t)(s_{\bar{x}})$	$\hat{p} \pm (z)(s_{\hat{p}})$
where:	*where:*
$t = t_{95\%}$ ***for 90%***	$z = 1.65$ ***for 90%***
$t = t_{97.5\%}$ ***for 95%***	$z = 1.96$ ***for 95%***
$t = t_{99.5\%}$ ***for 99%***	$z = 2.58$ ***for 99%***

Proportion Estimation Using the Calculator

To estimate a population proportion, p, from a sample $\hat{p}$,
use STAT → TESTS → 1-PropZInt

For example in a sample of n = 100 voters, x = 20 voters said they would vote the Independent candidate. At a 95% Confidence level, what proportion of the population of voters are expected to vote Independent?

```
1-PropZInt
 x:20         ← Independent voters in Sample
 n:100        ← Sample size
 C-Level:.95  ← Confidence Level
 Calculate
```

The result,

```
1-PropZInt
 (.1216,.2784) ← Prop Interval
 p̂=.2
 n=100
```

Thus the 95% confidence interval for the proportion of voters who we expect will vote Independent is between .1216 and .2784.

$.1216 < .2 < .2784$

$.2784 - .2 = .0784$ = Margin of Error

Estimation Using the Calculator Summary

<u>Estimate of the Population Mean (μ)</u>

<u>σ known</u> **<u>ZInterval (stats)</u>**
: pop std dev
: sample size
: sample mean
C-level: Confidence level

<u>Estimate of the Population Proportion (p)</u>

<u>1-PropZInt</u>
: no. of cases in sample
: sample size

C-level: Confidence level

<u>Estimate of the Population Mean (μ)</u>

<u>σ unknown</u> **<u>TInterval (stats)</u>**
: sample std dev
: sample size
: sample mean
C-level: Confidence level