

**8.1 Sampling Distribution of the Mean**  
Pg 363/Ex 7.1

A population of 7 students has ages  
23 19 20 21 18 19 25

The population mean ( $\mu$ ) = 20.71

Estimate the population mean by taking  
a random sample of 3 students...

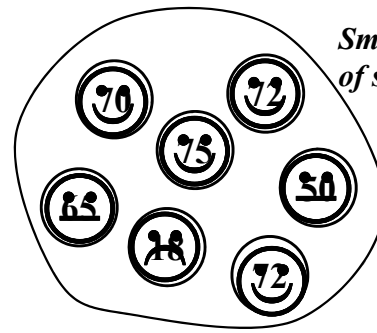
19 21 25

Find the sample mean...

$$\bar{X} = \frac{19+21+25}{3} = 21.67 \text{ but } \mu = 20.71$$

$$\text{Error} = \bar{X} - \mu = .96$$

**So...** we make an error when we estimate  
the population mean from a sample from that  
population.



Small population  
of student grades.

Population  
mean  
( $\mu$ ) = 60.3

Sample 2 students, calc sample mean

$$\bar{X} = \frac{18+72}{2} = 45, \text{ error} = 45 - 60.3 = 15.3$$

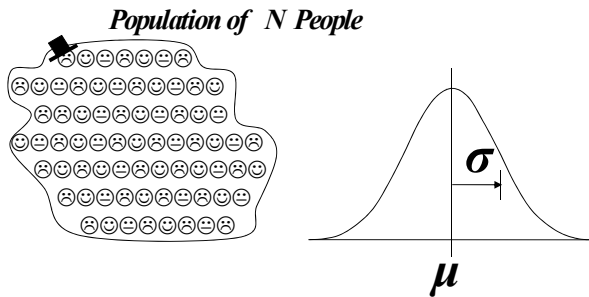
Sample 3 students, calc sample mean

$$\bar{X} = \frac{18+72+75}{3} = 55, \text{ error} = 55 - 60.3 = 5.3$$

Sample 4 students, calc sample mean

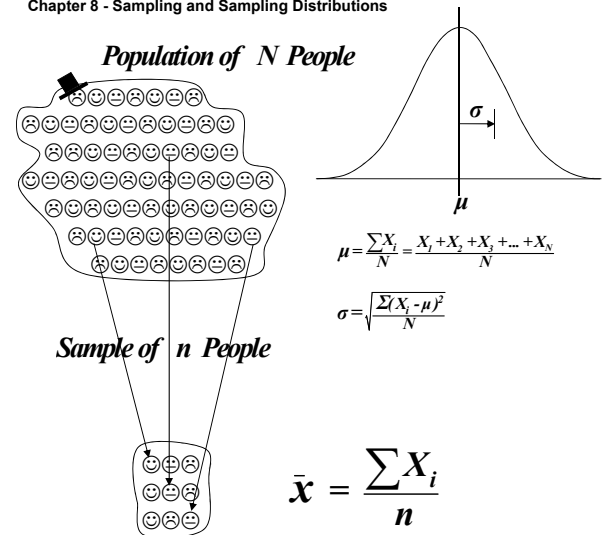
$$\bar{X} = \frac{18+72+75+65}{4} = 57.5, \text{ error} = 57.5 - 60.3 = 2.8$$

Calculate the mean  
and standard deviation of a population.



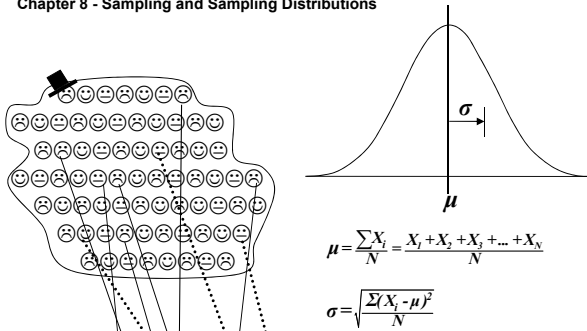
$$\mu = \frac{\sum X_i}{N} = \frac{X_1 + X_2 + X_3 + \dots + X_N}{N}$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$$



$$\bar{x} = \frac{\sum X_i}{n}$$

Calculate the mean value of the sample



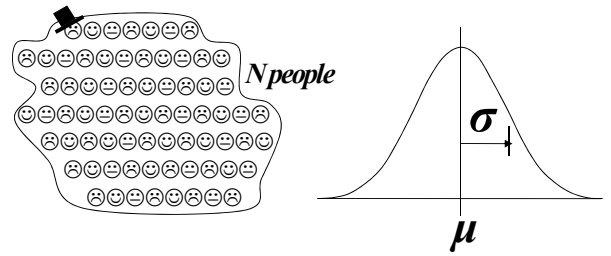
Take a number of samples of size  $n$

Sample 1:  $\bar{X}_1 = \frac{\sum X_i}{n}$

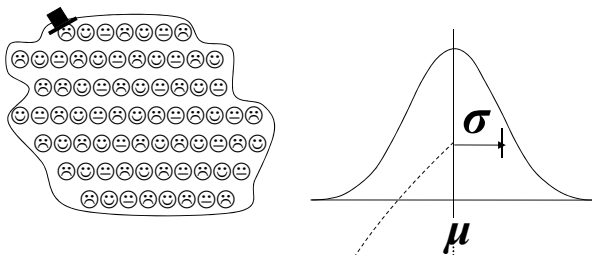
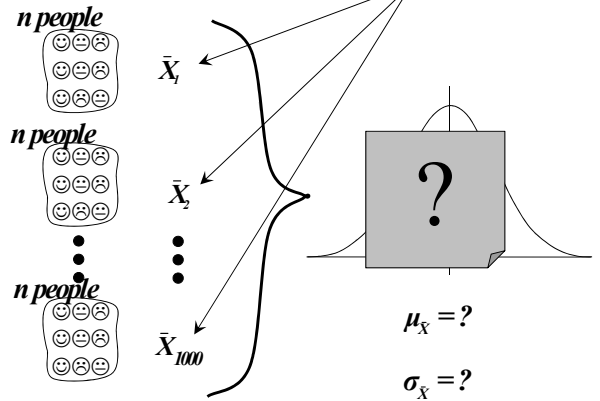
Sample 2:  $\bar{X}_2 = \frac{\sum X_i}{n}$

...many samples of size  $n$ ...

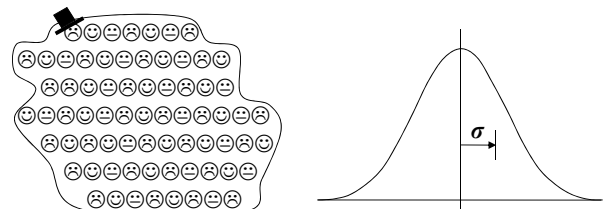
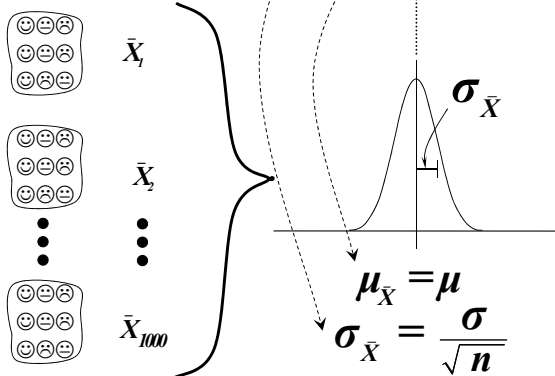
Sample 1000:  $\bar{X}_{1000} = \frac{\sum X_i}{n}$



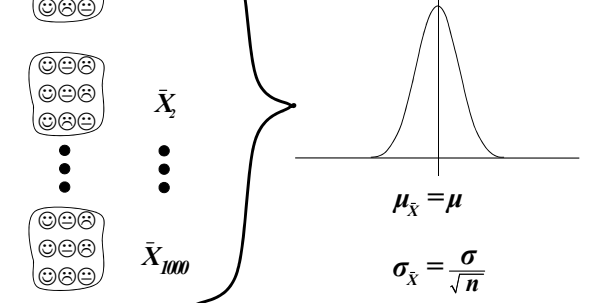
Make a distribution of the sample means.

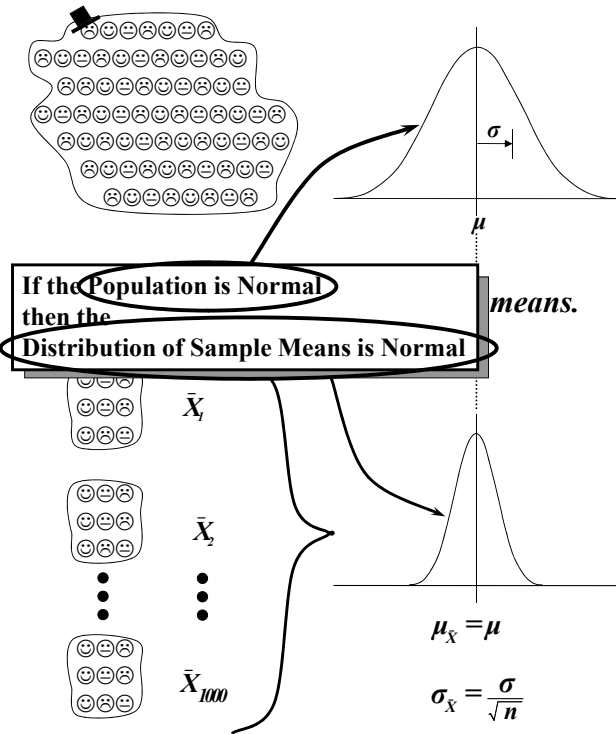


Make a distribution of the sample means.

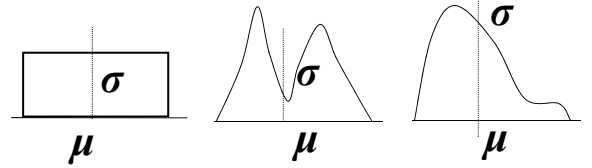


**NOTICE:**  
**IF THE POPULATION IS NORMAL**  
**THEN THE DISTRIBUTION OF SAMPLE MEANS IS NORMAL**



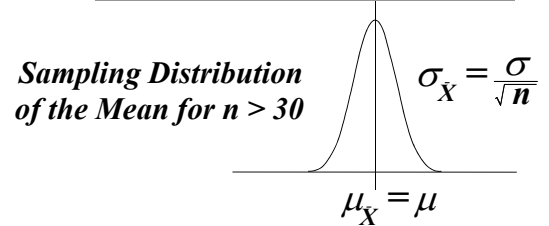


**Non-Normal Distributions of Populations**

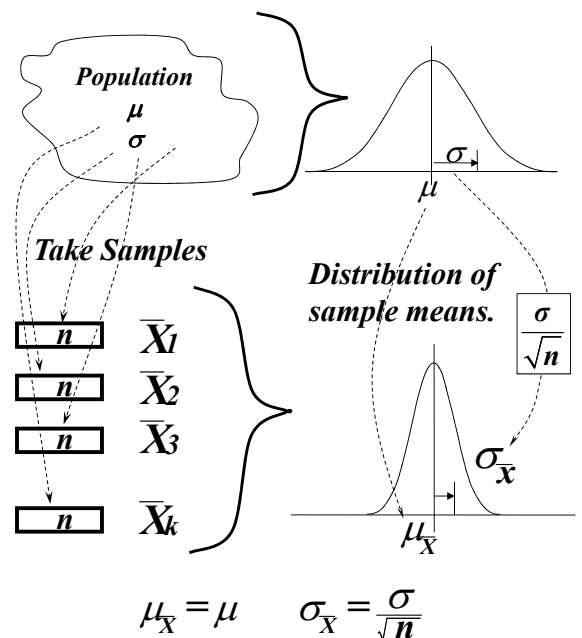
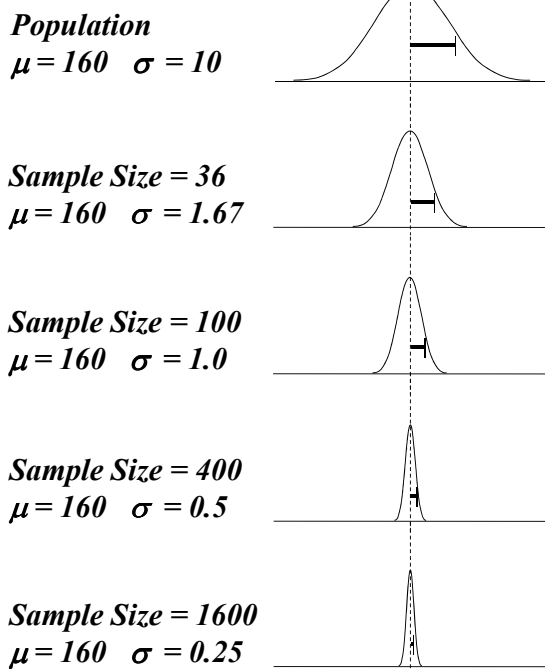


**ANOTHER NOTICE: Central Limit Theorem**

Regardless of the shape of the population distribution, the sample distribution of the mean approaches a Normal Distribution as the sample size n becomes large. ( Generally  $n > 30$  )



**Sampling Distribution of the Mean: Effect of Sample Size**



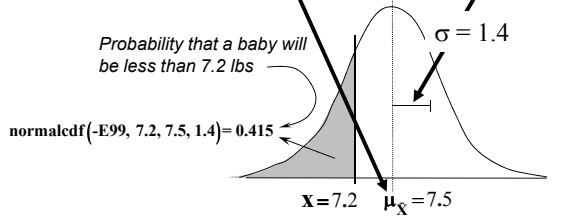
$\sigma_x$  is the Standard Error of the Mean

**Similar to Pg 473/Prob 39**

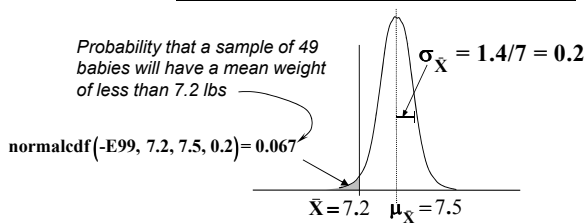
The mean weight of newborn babies in a Long Island community is 7.5 lbs. with a standard deviation of 1.4 pounds. What is the probability that

1. a baby will weigh < 7.2 lbs ?
2. a random sample of 49 babies will have a mean weight of < 7.2 lbs.?

**Distribution of Weights in Population**



**Distribution of Sample Mean Weights**



**The Life of a Car Battery**

A manufacturer of automobile batteries states that their "slow die" battery has a mean life of 50 months with a standard deviation of 6 months. If a consumer protection group randomly samples 49 of these batteries, what is the probability that the mean lifetime of the consumer group's sample will be less than 48 months (assuming the manufacturers claim is true)?

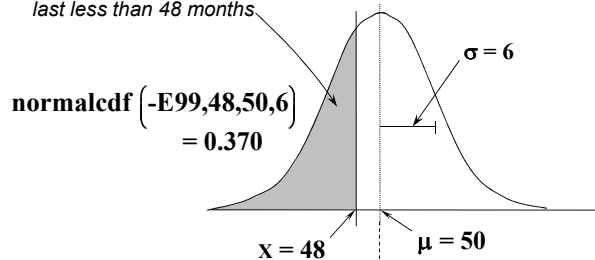
$\mu = 50$  months  
 $\sigma = 6$  months

Sample Size = 49

Looking for  
 Prob of sample means < 48 months

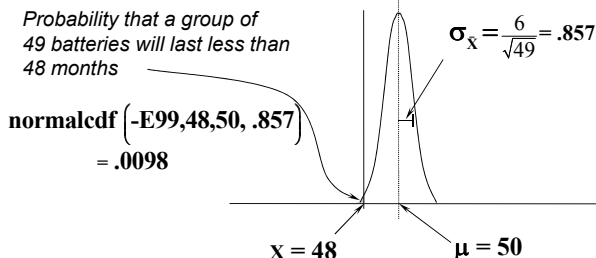
**Probability that a Single Battery will last < 48 Months**

Probability that a battery will last less than 48 months



**Probability that a Sample of 49 batteries will have a Mean Lifetime < 48 months**

Probability that a group of 49 batteries will last less than 48 months



**Section 7.7: Sampling Distribution of the Proportion**

A population of people can be composed of different groups (e.g democrats, Brooklynites, college graduates, etc.).

A population of items made by a manufacturer can be composed of different groups of items (items to sell at full price, items to sell at discount, items not saleable.)

A population of train arrival times can be composed of different categories ( Less than 1 minute late, more than 1 but less than 5 minutes late, etc.)

A proportion is the fraction equal to the number of items in one group divided by the total number of items.

Section 8.7: Sampling Distribution of the Proportion

**Proportion = Fraction of the population with a characteristic**

$p = X / N$  .... the true proportion

$X =$  occurrences in the population

$N =$  population

$\hat{p} = x / n =$  sample proportion

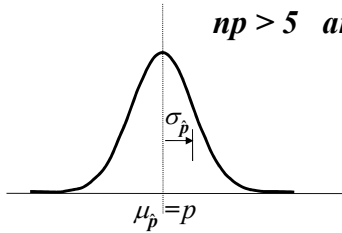
$x =$  occurrences in the sample

$n =$  sample size

**The Sampling Distribution of the Proportion**

can be approximated by a Normal Distribution if...

$np > 5$  and  $n(1-p) > 5$



$\mu_{\hat{p}} = p$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$  = standard error of the proportion

Example of a Proportion

Population	Sample
Nassau Community College 2001 - 2002	Sample of 50 Students from Nassau Community College
$N = 19,000$ Students	$n = 50$
Females: $X = 10,450$	Females: $x = 29$

$p =$  proportion of females

$\hat{p} =$  estim of  $p$

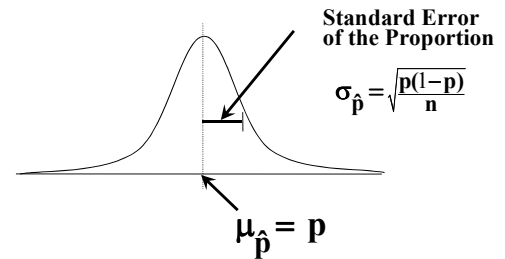
$p = \frac{X}{N}$

$\hat{p} = \frac{x}{n}$

Population Proportion =  $X / N$   
 $10,450 / 19,000 = .55$

Estimate of Population  
Proportion =  $x / n$   
 $29 / 50 = .58$

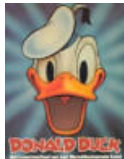
Sampling Distribution of the Proportion



Standard Error of the Proportion

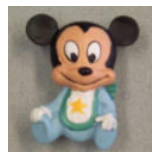
$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

Consider an Election



DD

$p = .53$



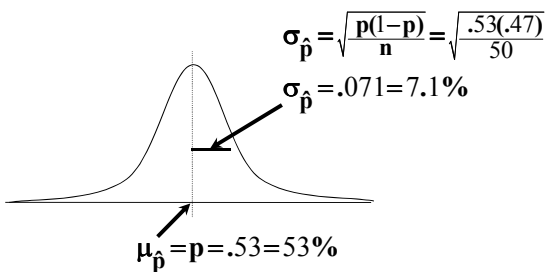
MM

$(1 - p) = .47$

Take a Sample of 50 voters ....  $n = 50$

What is the distribution of sample proportions for samples of size  $n = 50$ ?

Sampling Distribution of the Proportion for DD



$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.53(.47)}{50}}$

$\sigma_{\hat{p}} = .071 = 7.1\%$

$\mu_{\hat{p}} = p = .53 = 53\%$

Problems:

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