Toss a coin n times:

The probability of getting

s heads in n tosses is,

$$P(\text{s Head's in n tosses}) = {}_nC_s\, p^s (1-p)^{(n-s)}$$

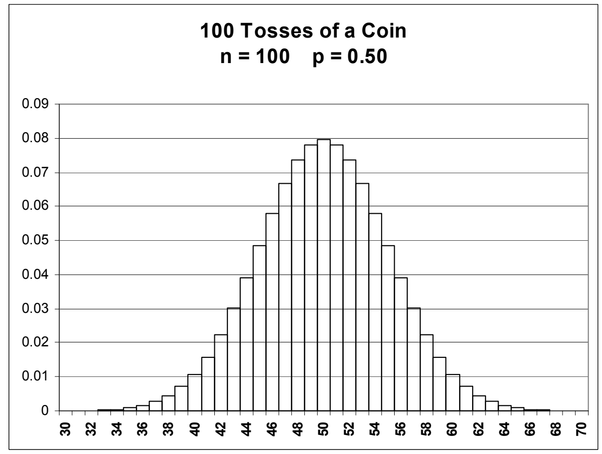
where,

n = number of tosses,

p = probability of a head,

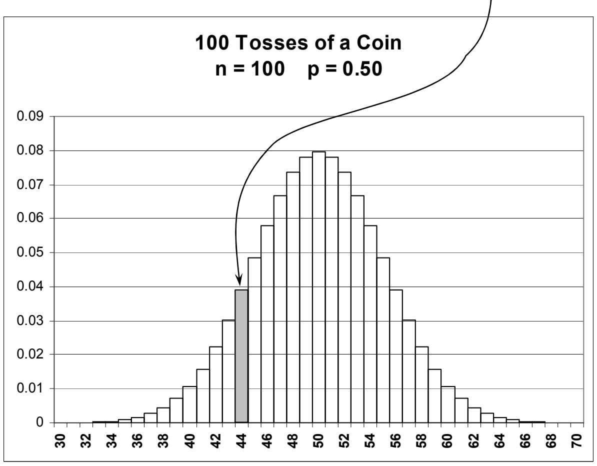
s = number of heads.

A graph (distribution) of these probabilities is below for n = 100 & p = 0.50



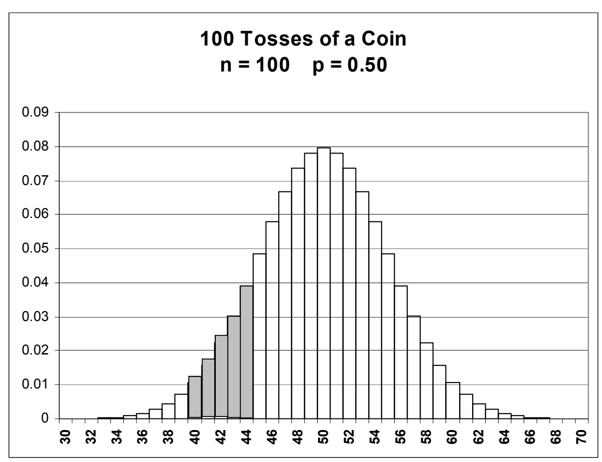
where the height of each bar in the graph is the probability of getting that many heads in 100 tosses of a coin.

For example, the probability of getting exactly 44 heads in 100 tosses is,



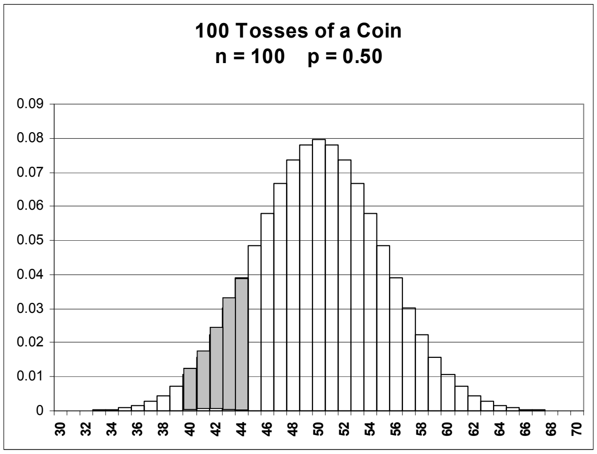
$$P(\text{44 Heads}) = {}_{100}C_{44}\, p^{44} (1-p)^{56}$$

The probability of getting between 40 and 44 heads in 100 tosses is the sum of the probabilities from 40 to 44.,



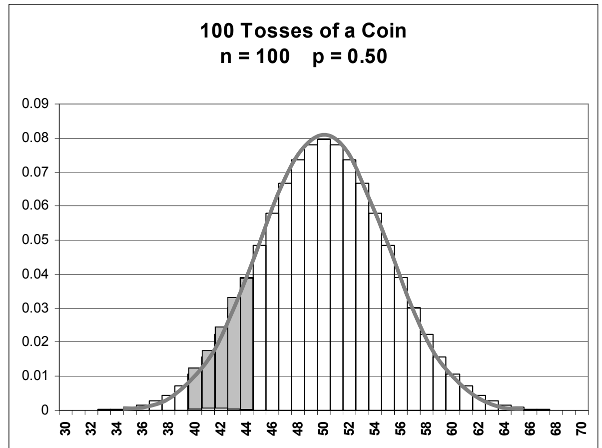
$$P(\text{40 to 44 Heads}) = {}_{100}C_{40}\, p^{40} (1-p)^{60} + {}_{100}C_{41}\, p^{41} (1-p)^{59} + {}_{100}C_{42}\, p^{42} (1-p)^{58} + {}_{100}C_{43}\, p^{43} (1-p)^{57} + {}_{100}C_{44}\, p^{44} (1-p)^{56}$$

Since each bar is one unit wide, the sum of the heights of the bars from 40 to 44 is just the area in grey.



And since <u>all the probabilities</u> must add up to 1, the area under all the bars must be 1.

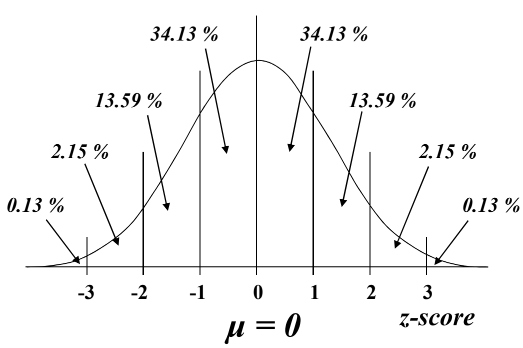
Notice that the shape of the binomial distribution of probabilities looks like The Normal Curve.



We can use the Normal Curve to approximate the area under the bars. That curve has

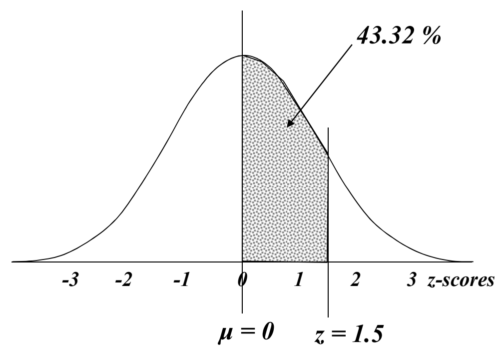
$$\mu = np = (100)(0.50) = 50 \quad \textit{and}$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{(100)(0.50)(1-0.50)} = 5$$

Recall, The Areas Under a Normal distribution for z-scores from -3 to +3 are ...



We need to look at how to find the areas under the normal curve <u>between any two z-scores</u>.

Finding <u>other</u> areas under the Normal curve ...



FIND: AREA BETWEEN $z_1 = 0$ and $z_2 = 1.5$

[2nd] [DISTR]

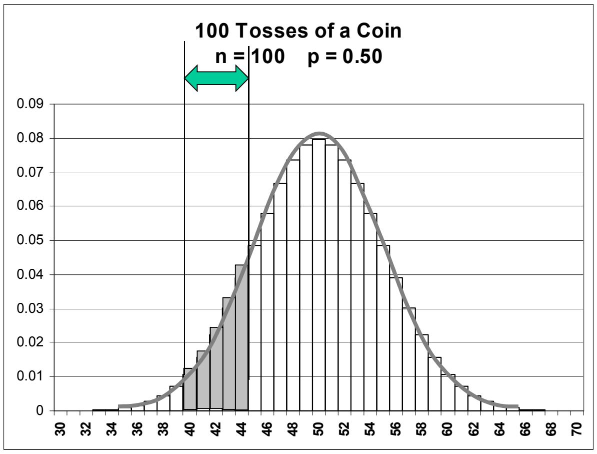
[2: normalcdf]

Left boundary *Right boundary*

normalcdf(0,1.5)

[ENTER]

Now we can use the area under the Normal curve to find the total area under bars 40 thru 44.

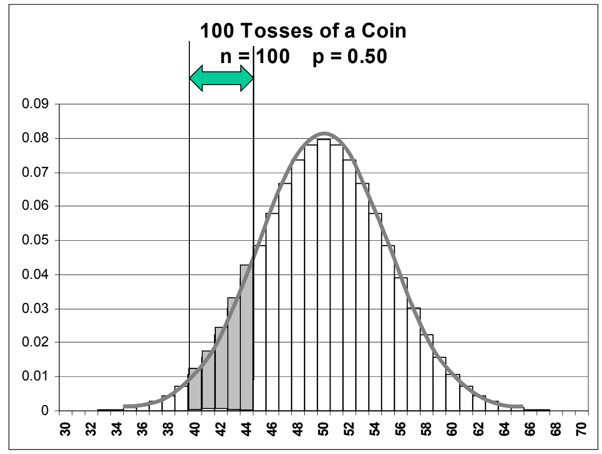


To use the calculator, we need to know the z-score of 39.5 and the z-score of 44.5. Recall, we need to do

$$Area = normalcdf(\, z_{39.5},\ z_{44.5})$$

Since the z-score of 39.5 is $z_{39.5} = \dfrac{39.5 - \mu}{\sigma}$

we need to know the mean, μ, and the standard deviation, σ, of the normal curve.

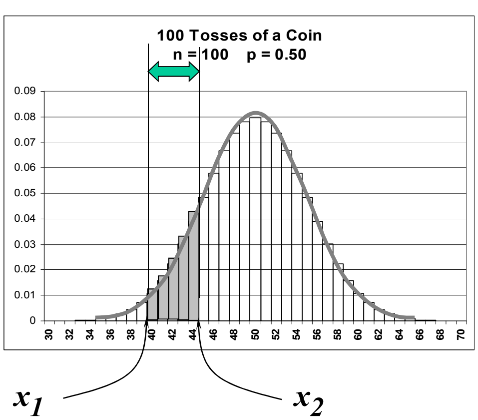


For a series of binomial trials,

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

For this Normal Curve, p = 0.5 and n = 100.
Then $\mu = np = (100)(0.5) = 50$ ***and***
$\sigma = \sqrt{np(1-p)} = \sqrt{(100)(0.5)(1-0.5)} = 5$

So to find the probability of getting between 40 and 44 Heads in 100 coin tosses, we find the area under the Normal Curve between $x_1 = 39.5$ ***and*** $x_2 = 44.5$.

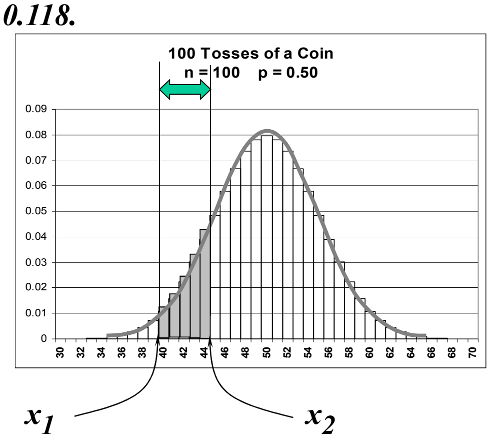


$$z_1 = \frac{39.5 - 50}{5} = \frac{-10.5}{5} = -2.1$$

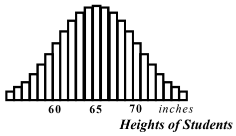
$$z_2 = \frac{44.5 - 50}{5} = \frac{-5.5}{5} = -1.1$$

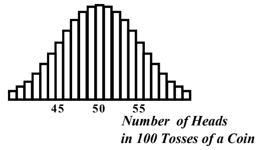
Area = normalcdf(- 2.1, -1.1) = 0.118

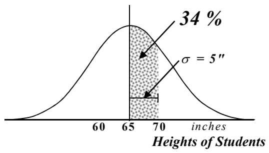
So the probability of getting between 40 and 44 heads in 100 flips of a fair coin is 0.118.

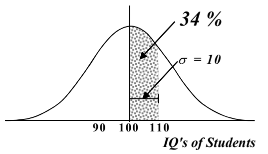


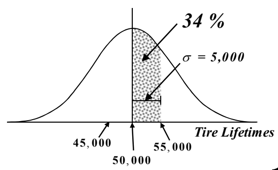
Applications of the Normal curve ...



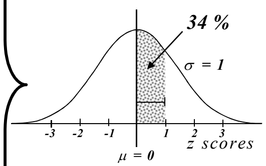






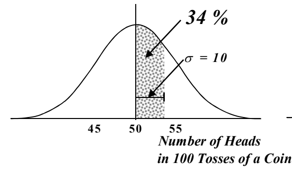


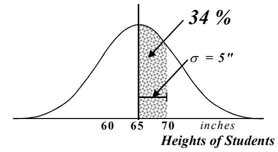
In all these examples, we may refer to the basic Normal Distribution with Mean = 0 and Std Dev = 1



34% is also the probability of finding a value from the population between the mean and 1 Std Dev above the mean.

Applications of the Normal curve ...





In the two example above, where the standard deviation is indicated, the significance of 34% is...

If you did the experiment of tossing a coin 100 times and you did that experiment over and over again, then, the percentage of times you would get between 50 and 55 heads is 34%.

That's another way of saying that the probability that you would get between 50 and 55 heads if you tossed a coin 100 times is 0.34.

If the height of a population of students were normally distributed with a mean of 65" and σ= 5", then, we would expect that 34% of the students would be between 65 and 70 inches tall.

That's another way of saying that the probability that a student you chose at random from this population would be between 65 and 70 inches tall would be 0.34.

Examples:

Section 7.4 Pg 410/12

Section 7.5 Pg 411/26, 30

Section 7.6 Pg 411/32, 34, 36

Section 7.7 Pg 412/38, 40, 42, 44

Section 7.8 Pg 412/46, 48, 50, 52

Section 7.9 Pg 413/60, 62, 64, 66, 68

Section 7.4 Pg 410/12

Draw a standard normal distribution from z = -4 to z = 4. On the normal curve, show the proportion of data values that fall between each integer z-score.