Random Variable: *a variable which can take on different numerical values which are determined by chance.*

Discrete Random Variable: *can take on a countable number of values.*

Continuous Random Variable: *can assume any value between two limits.*

Examples of
Discrete and Continuous Random Variables

The number of students in this room

The number of eye blinks a shopper makes in 1 minute

The temperature on the quad at noon

The number books in your room

The number of heads you get when you toss a coin 20 times

The time it takes to drive home

Your height

A Series of Trials

So now consider a sequence of trials (e.g. races).
Each trial has the probability of Success (S) or Failure (F).
Here is the outcome of your visit to the track.

S S F S F

or

Win Win Lose Win Lose

What is the probability of getting <u>exactly</u> this outcome sequence?

Call the probability of success, p.
Then the probability of failure is (1-p).

Since each outcome is independent of the others, we use the MULTIPLICATION RULE.

P(SSFSF) = pp(1-p) p(1-p)

A Series of Trials

P(SSFSF) = pp(1-p) p(1-p)

Thus if the probability of success in a single outcome were 1/8, (p = 1/8) then the probability of failure would be (1- 1/8) = 7/8

and the probability of getting the outcome
S S F S F

$= (1/8)\,(1/8)\,(7/8)\,(1/8)\,(7/8)$

$= (1/8)^3\,(7/8)^2 = 0.001495$

A Series of Trials

So the probability of getting exactly the sequence

S S F S F

is $(1/8)^3(7/8)^2 = 0.001495$

But what is the probability of getting

3 S's and 2 F's in any order?

We would need to make a list of all the possible ways we could get 3 S's and 2 F's in any order.

For example
S S F S F
S S F F S
S S S F F
S F F S S … and so on ….

A Series of Trials

So the probability of getting exactly the sequence

S S F S F

is $(1/8)^3(7/8)^2 = 0.001495$

But what is the probability of getting

3 S's and 2 F's in any order?

We would need to make a list of all the possible ways we could get 3 S's and 2 F's in any order. Each would occur with the probability $(1/8)^3(7/8)^2$.

Therefore, using the ADDITION RULE, we would add all of them up.

Thus P(3 S's & 2 F's)

$= (1/8)^3(7/8)^2$ x (No. of ways 3 S's & 2 F's can occur)

A Series of Trials

So what is the number of ways we can get a sequence with

3 S's out of 5 trials ?

There are exactly $\frac{5!}{(5-3)! \times 3!} = 10$

Then $(1/8)^3(7/8)^2 \times 10 = 10 \times 0.001495 = 0.01495$

which is the probability of getting 3 S's when we perform 5 trials and the probability of success in a single trial is 1/8.

The general formula for determining the number of ways we can get r S's out of n trials is abbreviated nCr, and is

$$nCr = \frac{n!}{(n-r)!\, r!)}$$

$$P(s \text{ in } n) = {}_nC_s\ (p)^s\ (1-p)^{n-s}$$

p = probability of success in a single trial
n = number of trials
s = number of successes

In general this applies to any Binomial experiment.

A binomial experiment satisfies 4 conditions.

1. ***The n trials are identical.***
2. ***The n trials are independent***
3. ***Each outcome is classified as a success or a failure.***
4. ***P(success) is the same for each trial.***

Use the calculator to evaluate the probability of getting 2 Heads in 3 tosses of an unfair coin, P(H) = 0.2

$$P(2 \text{ in } 3) = {}_3C_2\,(0.2)^2(1-0.2)^{3-2}$$

$$P(2 \text{ in } 3) = {}_3C_2\,(0.2)^2(0.8)^1$$

To calculate ${}_3C_2$ ...

First Key in the 3

Then MATH ◀

to get to the PRB Menu.

Select nCr

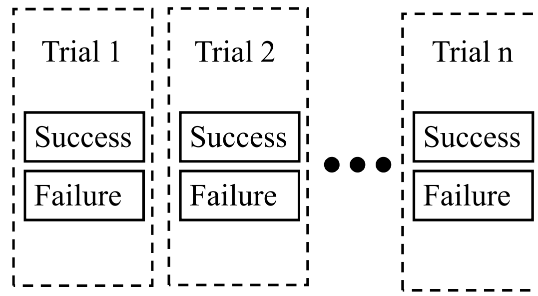
Then key in the 2

Then key in × 0.2 ^ 2 × 0.8 ^ 1

to see the result .096

$$P(s \text{ in } n) = {}_nC_s\,(p)^s\,(1-p)^{n-s}$$

p = probability of success in a single trial
n = number of trials
s = number of successes



P(Success in one trial) = p
P(Failure in one trial) = (1-p)

<u>Binomial Trial:</u>

1. n identical trials
2. n independent trials
3. Each outcome is success/failure
4. Prob(Success) is same in each trial.

$$P(s \text{ in } n) = {}_nC_s\,(p)^s\,(1-p)^{n-s}$$

p = probability of success in a single trial
n = number of trials
s = number of successes

Example: Pg 300/Ex 6.9

Toss a coin 5 times.

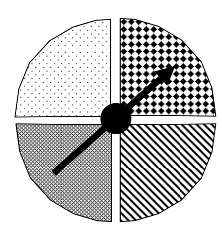
What is the probability of getting 0 Heads?

<u>Binomial Trial:</u>

1. n identical trials
2. n independent trials
3. Each outcome is success/failure
4. Prob(Success) is same in each trial.

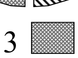
$$P(s \text{ in } n) = {}_nC_s\,(p)^s\,(1-p)^{n-s}$$

p = probability of success in a single trial
n = number of trials
s = number of successes



Example: Pg 301/Ex 6.10

Spin 4 times.

What is the probability of getting 3  ?

Binomial Trial:
1. n identical trials
2. n independent trials
3. Each outcome is success/failure
4. Prob(Success) is same in each trial.

$P(s \text{ in } n) = {}_nC_s\ (p)^s\ (1-p)^{n-s}$

p = probability of success in a single trial
n = number of trials
s = number of successes

Example: Pg 301/Ex 6.11

Medical records indicate that 40% of people in New York have type O+ blood.

Selecting 7 people at random what is the probability that 4 of them have type O+ ?

Binomial Trial:
1. n identical trials
2. n independent trials
3. Each outcome is success/failure
4. Prob(Success) is same in each trial.

$P(s \text{ in } n) = {}_nC_s\ (p)^s\ (1-p)^{n-s}$

p = probability of success in a single trial
n = number of trials
s = number of successes

Example: Pg 304/Ex 6.16

A student guesses at the answers to 5 questions on a multiple choice test. Each question has 4 choices.

1. What is the probability that the student will guess 3 answers correctly?
2. What is the probability that the student will guess 4 answers correctly?
3. What is the probability that the student will guess 3 or more answers correctly?

Problem Page 333/52

From past experience, Carol, a golfer knows that she will hit her drive into a sand trap from the green one-fourth of the time. Using this figure, what is the probability that she will hit the ball into the sand trap on exactly four of the next nine holes?

$P(s \text{ in } n) = {}_nC_s\ (p)^s\ (1-p)^{n-s}$

p = probability of success in a single trial
n = number of trials
s = number of successes

$P(s \text{ in } n) = {}_nC_s\ (p)^s\ (1-p)^{n-s}$

p = probability of success in a single trial
n = number of trials
s = number of successes

Similar Problem to Page 333/53

Medical records show that 40% of all persons affected with a certain viral illness recover.

A drug company has developed a new vaccine drug for this illness.

Ten people were selected at random and injected with the vaccine. Eight of the ten recovered shortly after injection.

1. Suppose that the vaccine were worthless: What is the probability that at least 8 people injected with the vaccine would recover?
2. Based on this, would you consider the vaccine helpful in recovery?

$$P(s \text{ in } n) = {}_nC_s\,(p)^s\,(1-p)^{n-s}$$

p = probability of success in a single trial
n = number of trials
s = number of successes

Problem Page 333/55

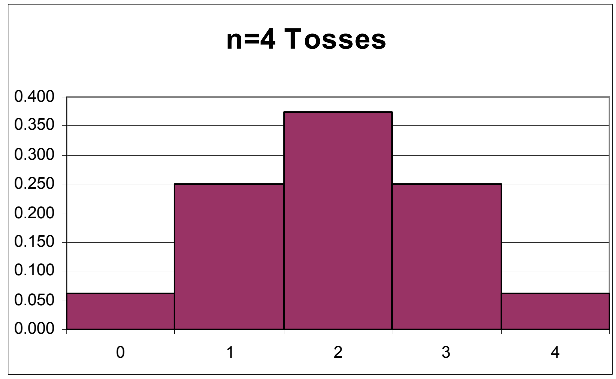
A slot machine has four windows. The wheels behind each window have the same 5 identical objects. They are cherry, grape, peach, star and jackpot. After the handle is pulled, the wheels revolve independently several times before coming to a stop. If the slot machine is played once, what is the probability that:

a) Four jackpots appear in the window?
b) At most 1 grape appears in the windows?
c) Less than 3 stars appear in the windows?
d) Only fruits appear in the windows?

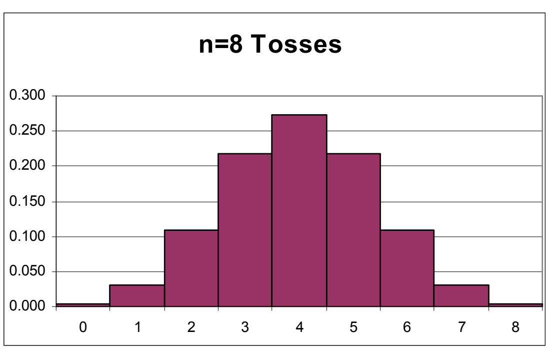
Toss a coin 4 times.
Plot the Probability of HEADS
For 2 HEADS …

$$P(2\ HD's) = {}_nC_s\, p^s q^{(n-s)} = {}_4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{(4-2)}$$

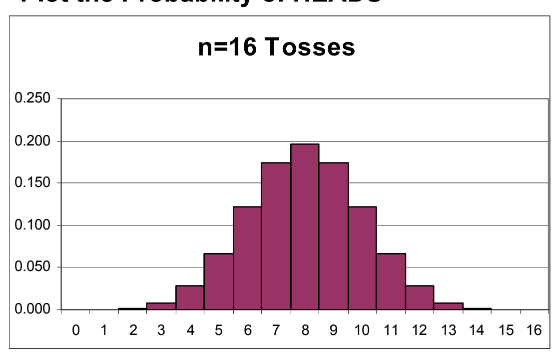
$$= (6)(.25)(.25) = .375$$



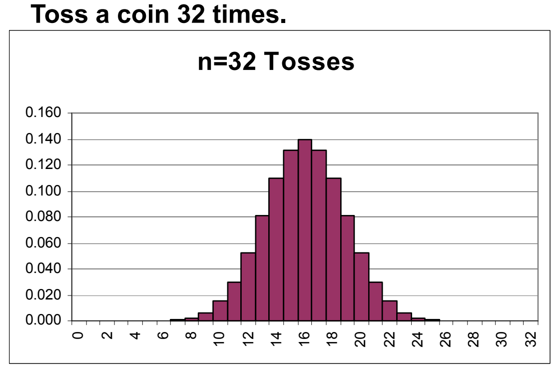
Toss a coin 8 times. Plot the Probability of HEADS



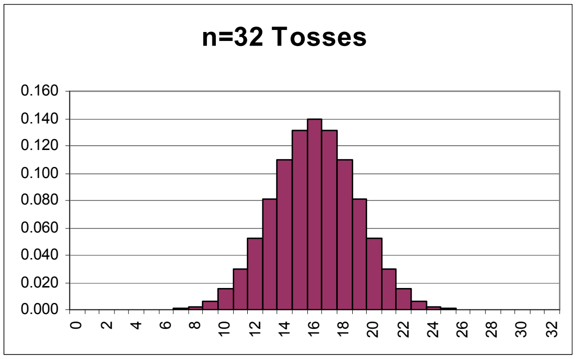
Toss a coin 16 times.
Plot the Probability of HEADS



Toss a coin 32 times.



Toss a coin 32 times. Plot the Probability of HEADS



Recall $n = 32,$ $p = 0.5$

The mean and standard deviation of the binomial distribution are

$\mu = np$ ***and*** $\sigma = \sqrt{np(1-p)}$