Experiment: ***A process in which an observation is made***

Betting on a horse

Selecting a card from a deck

Buying a lottery ticket

Tossing a coin

Sample Space: ***A representation of all the possible outcomes of an experiment.***

Head, Tail

A♣, 2♣, 3♣, 4♣,… … ,Q♥ ,K♥ (52 cards)

3.8 million possible lottery numbers

8 horses

Event: ***A particular collection of outcomes in the sample space of an experiment.***

Getting 1 Head in 2 tosses of a coin

Choosing any ♣ in a deck of 52 cards

Choosing any King in a deck of 52 cards

Throwing a die and getting at least a 4

Probability: ***is the likelihood that an event will occur.***

$0 \leq P \leq 1$

P = 0 ………The event can never occur

P = 1 ………The event will always occur

Determining Probability - Approaches:

Classical or a priori:
Determines probability by counting events that are equally likely.

Relative Frequency or a posteriori:
Determines probability by counting how often an event occurred.
(Law of Large Numbers)

Subjective or Personal:
Determines probability by personal judgement.

What is the probability of throwing 2 dice and getting a 7 or an 11?

List all the possible outcomes.
(This is called the "Sample Space.")

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

What is the probability of throwing 2 dice and getting a 7 or an 11?

How many outcomes are in the Sample Space? ***6 x 6 = 36***

How many outcomes total 7? ***6***

How many outcomes total 11? ***2***

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

$$\text{So } P(7or11) = \frac{6+2}{36} = \frac{8}{36} = 0.22$$

INDEPENDENT EVENTS

TOSS 2 FAIR COINS

(Fair means P(H) = P(T) = 1/2)

Possible Outcomes are **HH HT TH TT**

Each coin flip is an INDEPENDENT event which doesn't depend on the outcome of any other flip.

Then the probability of getting two heads is P(H) x P(H) = 1/2 x 1/2 = 1/4.

This is the <u>multiplication rule for independent events</u>.

If we toss three coins and ask the probability of getting HTT, we multiply P(H) x P(T) x P(T).

<u>Using the MULTIPLICATION RULE:</u>

Multiply the probabilities of <u>independent</u> events

Prob(HTT) = P(H) x P(T) x P(T)
Prob(HTT) = (1/2) (1/2) (1/2) = 1/8

MUTUALLY EXCLUSIVE EVENTS

Toss two fair coins: (4 outcomes, all equally likely)

coin 1	**2**	
H	**H**	**P(HH) = 1/2 x 1/2 = 1/4**
H	**T**	**P(HT) = 1/2 x 1/2 = 1/4**
T	**H**	**P(TH) = 1/2 x 1/2 = 1/4**
T	**T**	**P(TT) = 1/2 x 1/2 = 1/4**

What is the probability of at least 1 H?

If events are mutually exclusive, add their individual probabilities.

H H
H T
T H
T T

Add Probabilities of Mutually Exclusive Events.
P(HH) + P(HT) + P(TH) = 3/4

This is the <u>ADDITION RULE</u> for mutually exclusive events.

An Example:

Toss three fair coins: (8 outcomes, equally likely)

H H H	**Prob(HHH) = 1/8**
H H T	**Prob(HHT) = 1/8**
H T H	**Prob(HTH) = 1/8**
H T T	**Prob(HTT) = 1/8**
T H H	**Prob(THH) = 1/8**
T H T	**Prob(THT) = 1/8**
T T H	**Prob(TTH) = 1/8**
T T T	**Prob(TTT) = 1/8**

What is the probability of exactly 2 H's?

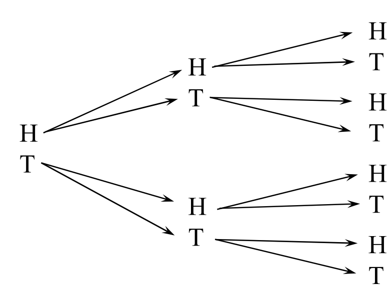
<u>Using the Addition Rule:</u>
Add probabilities of favorable events
Prob(2 H's) = P(HHT) + P(HTH) + P(THH)

Prob(2 H's) = 1/8 + 1/8 + 1/8 = 3/8 = 0.375

Prob(HHH) = 1/8	**HHH**
Prob(HHT) = 1/8	**HHT**
Prob(HTH) = 1/8	**HTH**
Prob(HTT) = 1/8	**HTT**
Prob(THH) = 1/8	**THH**
Prob(THT) = 1/8	**THT**
Prob(TTH) = 1/8	**TTH**
Prob(TTT) = 1/8	**TTT**

How did I determine all the possible outcomes? Easy!

1st outcome 2nd outcome 3rd outcome



Now just read across the rows!

MULTIPLICATION RULE:
In a trial consisting of independent events, the probability of 1 or more events is the product of the event probabilities.

…in other words…

MULTIPLICATION RULE:
In a trial consisting of independent events with outcomes, A, B, C, … etc.

the probability of getting an A and B and C, is
Prob of (A and B and C)
= Prob(A) × Prob(B) × Prob (C)

ADDITION RULE:
In a trial consisting of mutually exclusive, events, the probability of 1 or more events is the sum of the event probabilities.

…in other words…

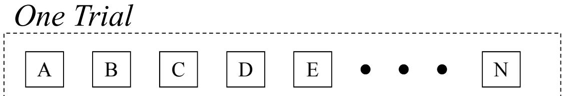
ADDITION RULE:
In a trial consisting of mutually exclusive outcomes, A, B, C, … etc.

the probability of getting an A or B or C, is
Prob of (A or B or C)
= Prob(A) + Prob(B) + Prob (C)

ADDITION RULE:

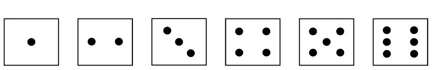
One Trial with different possible outcomes.
Outcomes are mutually exclusive.
(Only one can occur.)

One Trial



The probability that say A OR C OR D occurs, is $P_A+P_C+P_D$

Example: Throw a die.
Outcomes (sample space)

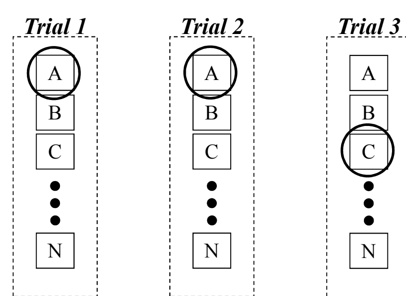


The probability that you get a 2, 3 or 5 is

$$P_2 + P_3 + P_5 = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$

MULTIPLICATION RULE:

Multiple Trials (say 3).
(Outcomes are independent of each other.)



The probability that, when you do all 3 trials, you get outcome AAC (A and A and C) is
$\textit{Prob of getting outcome AAC} = (P_A)\times(P_A)\times(P_C)$

Example: Throw a die 3 times.
Probability of getting sequence 3,4,5 is

$$Prob(3,4,5) = P_3 \times P_4 \times P_5 = \frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$$

Another Example:

Toss three coins

<u>What is the probability of getting 2 Heads in 3 tosses?</u>

But wait!…

Another Example:

Toss three coins

<u>What is the probability of getting 2 Heads in 3 tosses?</u>

But wait!… What if the probability of a H is not 0.5?
Suppose P(H) = 0.2 and P(T) = 0.8

Using the
<u>MULTIPLICATION RULE</u>

H H H
H H T → **P(HHT)= (0.2)(0.2)(0.8) = 0.032**
H T H → **P(HTH)= (0.2)(0.8)(0.2) = 0.032**
H T T
T H H → **P(THH)= (0.8)(0.2)(0.2) = 0.032**
T H T
T T H
T T T

Then using the
<u>ADDITION RULE</u>

Prob of 2 H's in 3 tosses is
the probability of HHT <u>or</u> HTH <u>or</u> THH

= P(HHT) + P(HTH) + P(THH)
= 0.032 + 0.032 + 0.032 = .096

SUMMARY:

Probability of an Event is between 0 and 1: $0 \le P(E) \le 1$

The sum of probabilities of all possible outcomes must be 1.
$P(A) + P(B) + \dots + P(Z) = 1.0$

If all outcomes are equally likely...

$$\textit{Probability of an Event} = \frac{\text{Number of outcomes satisfying event}}{\text{Total number of possible outcomes}}$$

Probability Rule: <u>Multiplication Rule</u> for Independent Events

If two events, A and B, are independent, (that is, the occurrence of one has no effect on the occurrence of the other) the probability that <u>event A and event B</u> will occur is the product of their probabilities.

P(A and B) = P(A) x P(B)

Probability Rule: Addition Rule for Mutually Exclusive Events

If two events, A and B, are mutually exclusive, the probability that <u>event A or event B</u> will occur is the sum of their probabilities.

P(A or B) = P(A) + P(B)

Simple Probability Problems

Pg 267/48
Determine whether this scenario is an example of classical, relative frequency, or subjective probability.
a) Allison does not know the probability of an unbalanced coin landing heads up. She determines that for 100 tosses of a coin, the frequency of heads is 65. She claims the probability that the unbalanced coin lands heads is 65/100
b) Pollsters from the Arizona Cactus claim that the probability of getting bitten by a rattlesnake is 1/100,000 for those hiking in the foothills of Tucson.
c) The probability that a homeless person survives for three years in a major city is 2/3.
d) Based on a sample of 150,000 high school seniors the probability that a high school senior basketball player makes a professional team is 1/2344.

Pg 267/51
If we toss a fair coin and roll a fair die at the same time, what is the probability of getting ...
a) a six on the die and a head on the coin.
b) an even number on the die and a head on the coin
c) a seven on the die and a tail on the coin.

Pg 267/52
find the probability of each of the following events.
a) Selecting the Ace of Spades by randomly choosing one card from a well-shuffled deck containing 52 cards
b) randomly selecting the lottery ticket numbered 0007, from a bin containing lottery tickets numbered 0000 to 9999.
c) guessing the month in which your teacher was born
d) randomly selecting an incorrect response to a 5 item multiple-choice question
e) randomly selecting a loser in and a horse race.

Simple Probability Problems

Pg 267/55
a) Construct a sample space for a four-child family. Assuming each outcome is equally likely, what is the probability of:
b) exactly one boy?
c) at least one boy?
d) at least two girls?
e) at most one boy?
f) no boy?

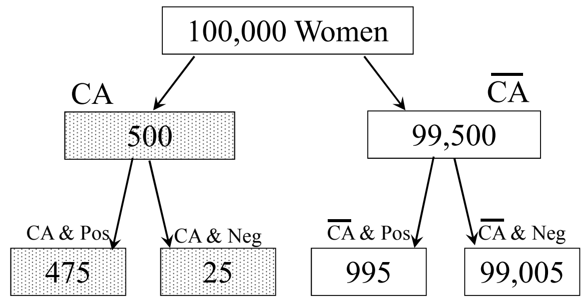
Pg 269/93
If a random drawing is performed and five letters are selected without replacement from the alphabet, determine the probability that:

a) the first three letters will spell the word YES.
b) the first five letters will spell the word GREAT.

Breast Cancer Screening

A member of your family is screened for Breast Cancer.
The test comes back Positive.
What is the probability that she actually has it?

Basic Information:
The incidence of breast cancer in the population of women is 0.5%.
(0.5% = 0.005, or about 1 in 200).
The Test:
If breast cancer is present,
there is a 95% probability the test will be positive.
If breast cancer is NOT present,
there is a 1% probability the test will be positive.



Given a positive result for breast cancer, the probability that the person actually has it is…

$$P(CA / PosTest) = \frac{475}{475+995} = 0.32 \approx 32\%$$

Simple Probability Problems

Four balls numbered 1 to 4 inclusive are placed in an urn. Randomly choose the four balls from the urn without replacement . Record the number appearing on each ball as it is selected so as to from a 4-digit number. What is the probability that:

a) The number 1234 is obtained?

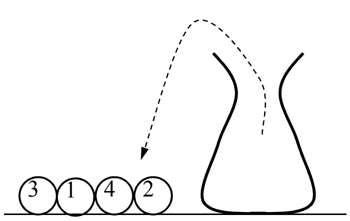
b) The number ends in a 4?

c) The number obtained is greater than 3200?

d) The number obtained is not greater than 4000?

e) The number obtained is either even or less than 3200?

f) The number obtained is odd and greater than 5000?



Simple Probability Problems

Pg 271/126

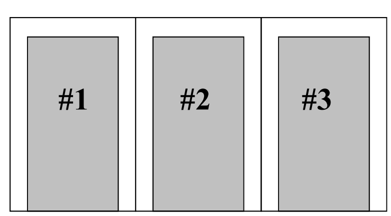
The Monte Hall Problem

On a game show you are given the choice of three doors. Behind one of the doors is a car and behind the other two doors are goats.

You select one of the doors

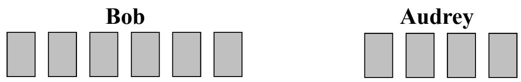
Mote Hall, the host, opens one of the other two doors and shows you that there is a goat behind it. Then he asks you if you now wish to change your choice of doors.

Should you change your selection?



Simple Probability Problems

A set of 10 cards contains two jokers. There are two players, Bob and Audrey. Bob chooses six cards at random and Audrey gets the remaining four cards. What is the probability that Bob has at least one joker?



Simple Probability Problems

Pg 269/102
In the game of chuck-a-luck a participant wagers on the sum of two die to be above or below 7.
a) What is the probability of the sum being below 7?
b) What is the probability of the sum not been below 7?

Pg 269/104
Lorenzo and Luigi play a game where they simultaneously exhibit their right hands with one, too, three or four fingers extended.
A) list all the possible outcomes in the sample space.
Determine the probability that:
B) both players extend the same number of fingers.
C) both players together expand and even number of fingers.
D) Lorenzo shows an odd number of fingers.